Goniometrické vzorce

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Definice McLaurinovým polynomem

$\sin x \equiv x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i+1}}{(2i+1)!}$
$\cos x \equiv 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i}}{(2i)!}$
$\arctan x \equiv x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots = \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i+1}}{2i+1}$
$\arcsin x \equiv x + \frac{x^3}{2\cdot 3} + \frac{1\cdot 3x^5}{2\cdot 4\cdot 5} + \frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\cdot 7} + \ldots = \sum_{i=0}^{\infty} \frac{(2i)! x^{2i+1}}{(2^i i!)^2 (2i+1)}$

Základní vztahy

Pythagorova věta $\sin^2 x + \cos^2 x = 1, 1 + \tan^2 x = \sec^2 x$.
Gaussova věta $e^{ix} = \cos x + i \sin x$.
Moivrova věta $(\cos x + i \sin x)^n = \cos nx + i \sin nx$.
Sudost a lichost funkcí $\sin x = -\sin(-x), \cos x = \cos(-x)$.
Periodičnost $\forall k \in \mathbb{Z} : \sin(x + 2k\pi) = \sin(x)$.
Definiční vztahy $\tan x \equiv \frac{\sin x}{\cos x}, \sec x \equiv \frac{1}{\cos x}, \csc x \equiv \frac{1}{\sin x}$.

Derivace a integrály

$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$
$(\sec x)' = \sec x \tan x$
$(\tan x)' = \sec^2 x$
$\int \sec x \, \mathrm{d}x = \ln|\sec x + \tan x| + C$
$\int \tan x \, \mathrm{d}x = C - \ln|\cos x|$
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
$(\arctan x)' = \frac{1}{1+x^2}$
$(\mathrm{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$

Sčítací vzorce

$\sin(x + y) = \sin x \cos y + \cos x \sin y$
$\cos(x + y) = \cos x \cos y - \sin x \sin y$
$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$
$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$
$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2}$
$\cos x \cos y = \frac{\cos(x-y) + \cos(x+y)}{2}$
$\sin x \cos y = \frac{\sin(x+y) - \sin(x-y)}{2}$

Dvojnásobný a poloviční argument

$\sin 2x = 2 \sin x \cos x$
$\cos 2x = \cos^2 x - \sin^2 x$
$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
$\sin^2 x = \frac{1 - \cos 2x}{2}$
$\cos^2 x = \frac{1 + \cos 2x}{2}$
$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$

Sumy

$\sum_{n=0}^{N-1} \cos nx = \frac{\sin \frac{Nx}{2}}{\sin \frac{x}{2}} \cos\left[(N-1)\frac{x}{2}\right]$
$\sum_{n=0}^{N-1} \sin nx = \frac{\sin \frac{Nx}{2}}{\sin \frac{x}{2}} \sin\left[(N-1)\frac{x}{2}\right]$
$\forall p \in (-1, 1) : \sum_{n=0}^{\infty} p^n \cos nx = \frac{1 - p\cos x}{1 - 2p\cos x + p^2}$,
$\sum_{n=0}^{\infty} p^n \sin nx = \frac{p \sin x}{1 - 2p\cos x + p^2}$