Throwing snowballs

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September 22, 2006

1 Problem

One man throws a snowball from the height y_0 with speed v_0 under the angle of $\theta = 40\,^\circ$. He hits the roof which is $d = 30\,\mathrm{m}$ from him and its height is $h = 7\,\mathrm{m}$ under the angle $\alpha = -20\,^\circ$.

2 Solution

2.1 Writing equations

Let us orientate our coordinate system with the axis y vertical and the axis x on the ground, so that the origin are the feet of the throwing man and the whole motion takes place in a plane. The appropriate equations are

$$\begin{aligned} y(t) &= tv_0 \sin\theta - \frac{1}{2}gt^2 + y_0 \\ x(t) &= tv_0 \cos\theta \Rightarrow t = \frac{x}{v_0 \cos\theta} \end{aligned} \tag{1}$$

2.2 Slope and parametric equations

We can write the equation for the slope of the curve $y(t)$ as

$$\frac{\mathrm{d}y(t)}{\mathrm{d}x(t)} = \frac{y'}{x'} = \frac{v_0 \sin\theta - gt}{v_0 \cos\theta} = \frac{v_0 \sin\theta - \frac{xg}{v_0 \cos\theta}}{v_0 \cos\theta} = \tan\theta - \frac{xg}{v_0^2 \cos^2\theta} \tag{2}$$

We know the value of slope $\frac{\mathrm{d}y}{\mathrm{d}x}|_{x=d} = \tan\alpha$. Plugging into (2) gives

$$\begin{aligned} \tan\theta - \frac{dg}{v_0^2 \cos^2\theta} &= \tan\alpha \\ v_0 &= \sqrt{\frac{dg}{\cos^2\theta\,(\tan\theta - \tan\alpha)}} \end{aligned} \tag{3}$$

To calculate the time is now easy; we just plug the result (3) to (1). From here hence

$$t = \sqrt{d\frac{\tan\theta - \tan\alpha}{g}} \tag{4}$$

The initial height is calculated from equation (1)

$$\begin{aligned} y_0 &= h - tv_0 \sin\theta + \frac{1}{2}gt^2 \\ &= h - d\tan\theta + d\frac{\tan\theta - \tan\alpha}{2} \\ &= h - d\frac{\tan\theta + \tan\alpha}{2} \end{aligned}$$

Thus the speed with which it hits the roof goes from the Conservation of energy as

$$\begin{aligned} v^2 + 2gh &= v_0^2 + 2gy_0 \\ v &= \sqrt{v_0^2 + 2g\left(y_0 - h\right)} \\ &= \sqrt{\frac{dg}{\cos^2\theta\left(\tan\theta - \tan\alpha\right)} - dg\left(\tan\theta + \tan\alpha\right)} \end{aligned}$$